Earthquake Engineering & Structural Dynamics
The Journal of the International Association for Earthquake Engineering

EARTHQUAKE ENGINEERING & STRUCTURAL DYNAMICS
Earthquake Engng Struct. Dyn. 2015; **44**:2241–2260
Published online 5 May 2015 in Wiley Online Library (wileyonlinelibrary.com). DOI: 10.1002/eqe.2581

Modal response analysis of multi-support structures using a random vibration approach

Ernesto Heredia-Zavoni[1,*,†], Sandra Santa-Cruz[2] and Francisco L. Silva-González[1]

[1] *Instituto Mexicano del Petróleo, Eje Central Lázaro Cárdenas 152, Col. San Bartolo Atepehuacan, México DF, 07730, Mexico*
[2] *Departamento de Ingeniería, Pontificia Universidad Católica del Perú, Av. Universitaria 1801, Lima, 32, Peru*

SUMMARY

A formulation is developed for modal response analysis of multi-support structures using a random vibration approach. The spectral moments of the structural response are rigorously decomposed into contributions from spectral moments of uncoupled modal responses. An advantage of the proposed formulation is that the total dynamic response can be obtained on the basis of mode by mode uncoupled analyses. The contributions to the total response from modal responses under individual support ground motions and under cross-correlated pairs of support ground motions can be recognized explicitly. The application and performance of the formulation is illustrated by means of an example using a well-established coherency spectrum model and widely known power spectra models, such as white noise and Kanai–Tajimi. The first three spectral moments of displacement, shear, and bending moment responses are computed, showing that the formulation produces the same results as the exact solution. Copyright © 2015 John Wiley & Sons, Ltd.

Received 3 December 2014; Revised 13 March 2015; Accepted 16 March 2015

KEY WORDS: ground motion spatial variation; multi-support structures; seismic response analysis; random vibration; coherency spectrum; response spectrum method

1. INTRODUCTION

Multi-support structures, such as lifelines, piping systems, long-span bridges, and viaducts, can be subjected to differential seismic ground motions at the supports. The influence of spatially varying earthquake ground motion on the seismic response of multi-support structures has been widely recognized [1–10]. In general, spatial variability of ground motions may result from scattering of waves in heterogeneous media, from the difference in arrival times of waves at different locations and from varying local soil conditions. These three components of spatial variability are commonly known as the incoherence, wave passage, and site response effects, respectively. For engineering purposes, the spatial variation of earthquake ground motions can be modeled in the frequency domain by means of the coherency spectrum. It describes the statistical dependence of earthquake ground motions at different locations in the frequency domain. Some functional forms have been proposed for the coherency spectrum of ground acceleration based on theoretical studies and statistical analysis of recordings from dense arrays [11–15].

The seismic response of multi-support structures can be computed using time history analysis. It requires that a set of ground motions properly correlated in space and time be specified at the supports. Several simulation techniques can be applied to generate spatially correlated time series of

*Correspondence to: E. Heredia-Zavoni, Instituto Mexicano del Petróleo, Eje Central Lázaro Cárdenas 152, Col. San Bartolo Atepehuacan, México DF, 07730, Mexico.
†E-mail: eheredia@imp.mx

earthquake ground motion [16–20]. Evidently, the response obtained from time history analysis is specific to the particular set of support ground motions used as input. To obtain statistically significant response measures, one would need to produce a sample of responses computed for a large ensemble of sets of support ground motions generated from a ground motion model that statistically characterizes the spatial variation features at the structure's site, which increases the amount of computational effort. Alternatively, an approach based on random vibration theory has the advantage that statistical measures such as the mean peak response can be obtained given the probabilistic modeling of the ground motion. A full random vibration formulation allows computing the auto-power spectrum of the response in terms of the cross-power spectra of the support ground motions. However, a standard practice in earthquake engineering is to characterize ground motions for structural analysis and design by means of response spectra rather than power spectra. Consequently, response spectrum methods based on random vibrations principles have been proposed for multi-support structures [21–24].

In this paper, a formulation is developed for modal response analysis of multi-support structures using a random vibration approach. It differs from existing methods in that the dynamic response is expressed as the exact sum of fully uncoupled SDOF modal responses accounting rigorously for modal response correlation and spatial variation of seismic ground motion. The formulation is based on the Complete Square Root of Sum of Squares (c-SRSS) modal combination rule [25] derived for the response analysis of multi-degree of freedom systems subject to single ground motions. First, we focus on the auto-power spectrum of the structural response and use the c-SRSS approach to uncouple modal response contributions. Next, the structural response is expressed as the sum of uncoupled modal responses to the support ground motions. Due consideration is given to the convergence analysis of spectral moments of modal responses. Examples are included illustrating the application and performance of the formulation to compute the spectral moments of displacement, shear, and bending moment responses, using well known models of power spectra, such as white noise and Kanai–Tajimi, and a well-established model for the ground motion coherency spectrum representing incoherence and wave-passage effects. Final comments and findings are then summarized in the conclusions.

2. DYNAMIC RESPONSE

Consider a multi-support linear structural system with n response degrees of freedom subjected to m support earthquake ground motions. Let $\omega_i, \zeta_i, i=1, \ldots, n$ denote the modal frequencies and critical damping ratios of the structure, respectively, and $u_k(t)$, $k=1, \ldots, m$ denote the ground displacement at the kth support. In general, a response of interest, $Z(t)$, for example, internal forces in a member, displacements at a node, or stress at a point, can be expressed as the sum of a pseudo static and a dynamic component [26],

$$Z(t) = \sum_{k=1}^{m} a_k u_k(t) + \sum_{k=1}^{m} \sum_{i=1}^{n} c_{ki}\, y_{ki}(t) \tag{1}$$

where y_{ki} is the modal displacement response of a SDOF modal oscillator with natural frequency ω_i and critical damping ratio ξ_i subjected to the kth support ground acceleration $\ddot{u}_k(t)$;

$$\ddot{y}_{ki} + 2\xi_i\omega_i\ \dot{y}_{ki} + \omega_i^2 y_{ki} = -\ddot{u}_k \tag{2}$$

and $a_k = q^T r_k$, $c_{ki} = q^T \phi_i \gamma_{ki}$ are the effective-influence coefficients and effective modal participation factors, respectively; r_k is the kth column of the influence matrix, ϕ_i is the modal shape, γ_{ki} is the modal participation factor, and q is the transfer vector relating nodal displacements to the response of interest. Suppose the ground motions at the supports of the structure are modeled as zero-mean jointly stationary random processes, and thus, the response in each mode of the structure is also stationary. This assumption is reasonable for modeling the seismic response of structures subject to ground motions with duration of strong phase longer than the fundamental period of the system. Let

DOI: 10.1002/eqe

$G_{kk}(\omega)$ denote the one-sided auto power spectrum of ground acceleration $\ddot{u}_k(t)$ and $H_i(\imath\omega)$ the modal transfer function,

$$H_i(\imath\omega) = -\frac{1}{\omega_i^2 - \omega^2 + 2\imath\xi_i\omega_i\omega}; \quad \imath = \sqrt{-1} \tag{3}$$

Using basic principles of random vibration theory, the one-sided power spectrum of the response can be derived from Equation (1),

$$\begin{aligned} G_{ZZ}(\omega) = {} & \sum_{k=1}^{m}\sum_{l=1}^{m} a_k a_l \frac{G_{kl}(\imath\omega)}{\omega^4} - 2\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n} a_k c_{li} H_i(-\imath\omega)\frac{G_{kl}(\imath\omega)}{\omega^2} \\ & + \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ki}c_{lj}H_i(\imath\omega)H_j(-\imath\omega)G_{kl}(\imath\omega) \end{aligned} \tag{4}$$

where $G_{kl}(\imath\omega)$ is the ground acceleration cross-power spectrum given by

$$G_{kl}(\imath\omega) = \gamma_{kl}(\imath\omega)\sqrt{G_{kk}(\omega)G_{ll}(\omega)} \tag{5}$$

and $\gamma_{kl}(\imath\omega)$ is the coherency spectrum; $G_{kl}(\imath\omega) = G_{kk}(\omega)$ for $k = l$. The real and imaginary parts of $G_{kl}(\imath\omega)$ are known as the co-spectrum and quadrature spectrum, which are even and odd functions of frequency, respectively. The double and quadruple sums in Equation (4) represent the contributions from the pseudo-static and dynamic response, respectively; the triple sum accounts for the contribution from the cross-correlation between them. The power spectrum of the dynamic response includes contributions from cross-correlations between ground motions and cross-correlations between modal responses. Considering that $G_{kl}(\imath\omega) = G^*_{lk}(\imath\omega)$ and $[H_i(\omega)H_j(-\omega)] = [H_j(\omega)H_i(-\omega)]^*$, where Z^* denotes the complex conjugate of Z, the auto-power spectrum of the dynamic response, $G_{dd}(\omega)$, can be written as follows,

$$\begin{aligned} G_{dd}(\omega) = {} & \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ki}c_{lj}\mathrm{Re}\left[H_i(\omega)H_j(-\omega)\right]\mathrm{Re}G_{kl}(\omega) \\ & - \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ki}c_{lj}\mathrm{Im}\left[H_i(\omega)H_j(-\omega)\right]\mathrm{Im}G_{kl}(\omega) \end{aligned} \tag{6}$$

where Re and Im denote the real and imaginary parts, respectively. Heredia-Zavoni [25] derived the complete SRSS modal combination rule (c-SRSS) for the response analysis of structures subjected to a single earthquake ground motion, expressing $\mathrm{Re}[H_i(\omega)H_j(-\omega)]$ in terms of the squared norms $|H_i(\omega)|^2$ and $|H_j(\omega)|^2$ using a partial fractions expansion,

$$\mathrm{Re}\left[H_i(\omega)H_j(-\omega)\right] = A_{ij}|H_i(\omega)|^2 + B_{ij}\frac{\omega^2}{\omega_i^2}|H_i(\omega)|^2 + D_{ij}\left|H_j(\omega)\right|^2 + E_{ij}\frac{\omega^2}{\omega_j^2}\left|H_j(\omega)\right|^2 \tag{7}$$

The partial fraction factors A_{ij}, B_{ij}, D_{ij}, and E_{ij} only depend on the modal frequencies and the critical damping ratios; in terms of ratio $r = \frac{\omega_i}{\omega_j}$, they are given by

$$E_{ij} = \frac{\left[4\xi_j r\left(\xi_i - \xi_j r\right) + r^2 - 1\right](1 - r^4) - 2r^2(1 - r^2)\left[r^2\left(1 - 2\xi_j^2\right) - \left(1 - 2\xi_i^2\right)\right]}{4r^2\left[\left(1 - 2\xi_i^2\right)r^2 - \left(1 - 2\xi_j^2\right)\right]\left[\left(1 - 2\xi_j^2\right)r^2 - \left(1 - 2\xi_i^2\right)\right] - (1 - r^4)^2} \tag{8a}$$

$$B_{ij} = -r^2 E_{ij} \tag{8b}$$

$$D_{ij} = \frac{4\xi_i\xi_j r - r^2 - 1 + 2r^2\left(1 - 2\xi_j^2\right) - (r^4 - 1)E_{ij}}{2r^2\left[r^2\left(1 - 2\xi_j^2\right) - \left(1 - 2\xi_i^2\right)\right]} \tag{8c}$$

$$A_{ij} = r^2 - r^4 D_{ij} \tag{8d}$$

DOI: 10.1002/eqe

and satisfy $A_{ij}=D_{ji}$, $B_{ij}=E_{ji}$; for $r=1$, $A_{ij}=0.5, B_{ij}=0$. Using the partial fractions expansion, we can show that

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ki}c_{lj}\mathrm{Re}\left[H_i(\omega)H_j(-\omega)\right] = \sum_{i=1}^{n}\sum_{j=1}^{n}\left\{\left(c_{ki}c_{lj}+c_{kj}c_{li}\right)A_{ij}+\left(c_{ki}c_{lj}+c_{kj}c_{li}\right)\left(\frac{\omega}{\omega_i}\right)^2 B_{ij}\right\}\left|H_i(\omega)\right|^2 \quad (9)$$

Let us define the response coefficients $\alpha_{ikl}=\sum_{j=1}^{n}\left(c_{ki}c_{lj}+c_{li}c_{kj}\right)A_{ij}$, $\beta_{ikl}=\sum_{\substack{j=1\\ j\neq i}}^{n}\left(c_{ki}c_{lj}+c_{li}c_{kj}\right)B_{ij}$; then,

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ki}c_{lj}\mathrm{Re}\left[H_i(\omega)H_j(-\omega)\right] = \sum_{i=1}^{n}\left\{\alpha_{kli}+\beta_{kli}\left(\frac{\omega}{\omega_i}\right)^2\right\}\left|H_i(\omega)\right|^2 \quad (10)$$

For the imaginary part, $\mathrm{Im}[H_i(\omega)H_j(-\omega)]$, Heredia-Zavoni and Vanmarcke [22] developed the following partial fraction decomposition:

$$\mathrm{Im}\left[H_i(\omega)H_j(-\omega)\right] = A_{ij}^{'}\frac{\omega}{\omega_i}\left|H_i(\omega)\right|^2 + B_{ij}^{'}\frac{\omega^3}{\omega_i^3}\left|H_i(\omega)\right|^2 + D_{ij}^{'}\frac{\omega}{\omega_j}\left|H_j(\omega)\right|^2 + E_{ij}^{'}\frac{\omega^3}{\omega_j^3}\left|H_j(\omega)\right|^2 \quad (11)$$

Here, we express factors $A_{ij}^{'}, B_{ij}^{'}, D_{ij}^{'}$, and $E_{ij}^{'}$ in terms of ratio $r=\frac{\omega_i}{\omega_j}$ for consistency with Equation (8); thus, in the following expressions, these factors differ from the way they were originally written by Heredia-Zavoni and Vanmarcke [22] in terms of ratio ω_j/ω_i,

$$\begin{aligned} E_{ij}^{'} = & \frac{\left(1-r^4\right)\left[2\xi_i r-2\xi_j-2\left(2\xi_j r^2-2\xi_i r\right)\left(2\xi_j^2-1\right)\right]}{-\left(1-r^4\right)^2-\left[2r^2\left(2\xi_i^2-1\right)-2\left(2\xi_j^2-1\right)\right]\left[2r^2\left(2\xi_i^2-1\right)-2r^4\left(2\xi_j^2-1\right)\right]} \\ & +\frac{-\left(-2\xi_j r^2+2\xi_i r\right)\left(2r^2\left(2\xi_i^2-1\right)-2r^4\left(2\xi_j^2-1\right)\right)}{-\left(1-r^4\right)^2-\left[2r^2\left(2\xi_i^2-1\right)-2\left(2\xi_j^2-1\right)\right]\left[2r^2\left(2\xi_i^2-1\right)-2r^4\left(2\xi_j^2-1\right)\right]} \end{aligned} \quad (12a)$$

$$B_{ij}^{'} = -r^3 E_{ij}^{'} \quad (12b)$$

$$D_{ij}^{'} = \frac{2\xi_i r-2\xi_j r^2-E_{ij}\left[2r^2\left(2\xi_i^2-1\right)-2\left(2\xi_j^2-1\right)\right]}{1-r^4} \quad (12c)$$

$$A_{ij}^{'} = r^2\left(2\xi_j r-2\xi_i-r^3 D_{ij}\right) \quad (12d)$$

These factors depend only on the modal frequencies and critical damping ratios and satisfy $A_{ij}^{'}=-D_{ji}^{'}$, $B_{ij}^{'}=-E_{ji}^{'}$. Let the response coefficients $\alpha_{ikl}^{'}=\sum_{j=1}^{n}\left(c_{ki}c_{lj}-c_{li}c_{kj}\right)A_{ij}^{'}$ and $\beta_{ikl}^{'}=\sum_{j=1}^{n}\left(c_{ki}c_{lj}-c_{li}c_{kj}\right)B_{ij}^{'}$. Similarly to the aforementioned derivation, it can be shown that

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ki}c_{lj}\mathrm{Im}\left[H_i(\omega)H_j(-\omega)\right] = \sum_{i=1}^{n}\left\{\alpha_{kli}^{'}\left(\frac{\omega}{\omega_i}\right)+\beta_{kli}^{'}\left(\frac{\omega}{\omega_i}\right)^3\right\}\left|H_i(\omega)\right|^2 \quad (13)$$

DOI: 10.1002/eqe

Upon substitution of Equation (10) and (13) in Equation (6), we obtain

$$G_{dd}(\omega) = \sum_{i=1}^{n}\sum_{k=1}^{m}\sum_{l=1}^{m}\left[\alpha_{ikl} + \beta_{ikl}\left(\frac{\omega}{\omega_i}\right)^2\right]|H_i(\omega)|^2\mathrm{Re}G_{kl}(\omega) - \left[\alpha'_{ikl}\left(\frac{\omega}{\omega_i}\right) + \beta'_{ikl}\left(\frac{\omega}{\omega_i}\right)^3\right]|H_i(\omega)|^2\mathrm{Im}G_{kl}(\omega) \tag{14}$$

As shown in Equation (14), the power spectrum of the dynamic response can be expressed as a sum of uncoupled modal contributions.

3. SPECTRAL MOMENTS OF DYNAMIC RESPONSE

Let λ_q denote the qth-order spectral moment of the dynamic response

$$\lambda_q = \int_0^\infty \omega^q G_{dd}(\omega)\mathrm{d}\omega \tag{15}$$

For engineering purposes, the response can be characterized in terms of the first three spectral moments, λ_0, λ_1, and λ_2; λ_0 is the response variance, and λ_2 is the variance of its derivative. If the response is Gaussian, the square root $\sqrt{\lambda_2/\lambda_1}$ is the mean frequency of the response, and $\delta = \sqrt{1-\lambda_1^2/(\lambda_0\lambda_2)}$ is a bandwidth measure of the response power spectrum. The statistics and probability distribution of the maximum response in a time window are defined in terms of λ_0, λ_1, and λ_2. Substituting Equation (14) into Equation (15),

$$\begin{aligned}\lambda_q = \int_0^\infty \sum_{i=1}^{n}\sum_{k=1}^{m}\sum_{l=1}^{m}\Bigg\{&\left[\alpha_{ikl} + \beta_{ikl}\left(\frac{\omega}{\omega_i}\right)^2\right]\omega^q|H_i(\omega)|^2\mathrm{Re}G_{kl}(\omega)\\ &-\left[\alpha'_{ikl}\left(\frac{\omega}{\omega_i}\right) + \beta'_{ikl}\left(\frac{\omega}{\omega_i}\right)^3\right]\omega^q|H_i(\omega)|^2\mathrm{Im}G_{kl}(\omega)\Bigg\}d\omega\end{aligned} \tag{16}$$

which under proper convergence assurance can be written as

$$\lambda_q = \sum_{i=1}^{n}\sum_{k=1}^{m}\sum_{l=1}^{m}\left[\alpha_{ikl}\mathrm{Re}\lambda_{q,ikl} + \frac{\beta_{ikl}}{\omega_i^2}\mathrm{Re}\lambda_{q+2,ikl} - \frac{\alpha'_{ikl}}{\omega_i}\mathrm{Im}\lambda_{q+1,ikl} - \frac{\beta'_{ikl}}{\omega_i^3}\mathrm{Im}\lambda_{q+3,ikl}\right] \tag{17}$$

where

$$\lambda_{q,ikl} = \int_0^\infty \omega^q|H_i(\omega)|^2 G_{kl}(\omega)\mathrm{d}\omega \tag{18}$$

are modal spectral moments. Noting that for $k=l$, Im $\lambda_{q+1,ikk}$ = Im $\lambda_{q+3,ikk} = 0$,

$$\begin{aligned}\lambda_q = &\sum_{i=1}^{n}\sum_{k=1}^{m}\left[\alpha_{ikk}\lambda_{q,ikk} + \frac{\beta_{ikk}}{\omega_i^2}\lambda_{q+2,ikk}\right]\\ &+\sum_{i=1}^{n}\sum_{k=1}^{m}\sum_{\substack{l=1\\k\neq l}}^{m}\left[\alpha_{ikl}\mathrm{Re}\lambda_{q,ikl} + \frac{\beta_{ikl}}{\omega_i^2}\mathrm{Re}\lambda_{q+2,ikl} - \frac{\alpha'_{ikl}}{\omega_i}\mathrm{Im}\lambda_{q+1,ikl} - \frac{\beta'_{ikl}}{\omega_i^3}\mathrm{Im}\lambda_{q+3,ikl}\right]\end{aligned} \tag{19}$$

Interchange of summations and integration in Equation (16) to obtain Equation (17) is possible if the integrals in Equation (18) converge. If the auto-power spectra $G_{kk}(\omega)$ are band-limited, so that $G_{kk}(\omega) = 0$ for ω greater than some cutoff frequency, $\omega \geq \omega_f$, then these integrals are finite and the spectral moments in the right hand side of Equation (17) exist. For other

DOI: 10.1002/eqe

models of auto-power spectra, convergence must be analyzed. It is considered that the coherency function decays exponentially to zero, as will be discussed in Section 5, at a rate sufficient for $\lambda_{q+3,ikl}$ to converge. Therefore, when $k=l$, convergence of the integrals involving the auto-power spectrum, $G_{kk}(\omega)$, should be assured for the existence of the spectral moments of the response. For these reasons, the critical spectral moment in the right hand side of Equation (19) is

$$\lambda_{q+2,ikk} = \int_0^\infty \omega^{q+2}|H_i(\omega)|^2 G_{kk}(\omega)\mathrm{d}\omega \tag{20}$$

which exists if and only if the integrand is of order $\frac{1}{\omega^p}, p>1$, for large ω. Given that $\omega^{q+2}|H_i(\omega)|^2 \rightarrow \frac{1}{\omega^{2-q}}$ for large ω, the integral converges only if $G_{kk}(\omega)$ is of order $\frac{1}{\omega^p}, p>q-1$. So, for instance, if ground motion is modeled as white noise, the integral in Equation (20) converges only for $q<1$, and in such case, only the zeroth spectral moment of the structural response, λ_0, can be computed. However, if we consider the response power spectrum given in Equation (6), then, provided that $\mathrm{Re}\left[\omega^q H_i(\imath\omega)H_j(-\imath\omega)\right] \rightarrow \frac{1}{\omega^{4-q}}$ for large ω, convergence of the qth spectral moment of the response requires that $G_{kk}(\omega)$ only be of order $\frac{1}{\omega^p}, p>q-3$. Thus, in case of white noise excitation, spectral moments exist for $m<3$, and λ_0, λ_1, and λ_2 can be obtained. This issue has been analyzed by Igusa *et al.* [27] for the modal decomposition of the response of non-classically damped systems to a base acceleration. They solve the convergence problem introducing a modified spectral density, which removes the dominant term at large frequencies from the integrands of the spectral moments; the dominant term is such that it vanishes when summing up over all modal responses. Based on the solution proposed by Igusa *et al.* [27], we define a modified spectral density $G'_{ik}(\omega)$ as follows,

$$G'_{ik}(\omega) = |H_i(\omega)|^2 G_{kk}(\omega) - \frac{G_{kk}(\omega)}{\omega^4} \tag{21}$$

It can be shown that for large ω,

$$G'_{ik}(\omega) \rightarrow \frac{\left(1-2\zeta_i^2\right)2\omega_i^2}{\omega^6} G_{kk}(\omega) \tag{22}$$

Therefore, $\omega^{q+2}G'_{ik}(\omega)$ is of order $\frac{G_{kk}(\omega)}{\omega^{4-q}}$ and convergence requires that $G_{kk}(\omega)$ be of order $\frac{1}{\omega^p}, p>q-3$, for large ω. So now, for the case of white noise excitation, integrands of the type $\omega^{q+2}G'_{ik}(\omega)$ converge for $q<3$, and all three spectral moments λ_0, λ_1, and λ_2 can be obtained as well. The second component in Equation (21), $\frac{G_{kk}(\omega)}{\omega^4}$, only depends on the ground motion. Because, in a general case, this second component may diverge as $\omega \rightarrow 0$, the following definition is adopted from Igusa *et al.* [27] for the modified spectral density $G'_{ik}(\omega)$,

$$G'_{ik}(\omega) = \begin{cases} |H_i(\omega)|^2 G_{kk}(\omega) - \dfrac{G_{kk}(\omega)}{\omega^4}, & \omega \geq \omega_o \\ |H_i(\omega)|^2 G_{kk}(\omega), & 0<\omega<\omega_o \end{cases} \tag{23}$$

with ω_o being a fixed arbitrary positive frequency. In terms of the modified spectral density, $G'_{ik}(\omega)$, we define a modified spectral moment $\lambda'_{q+2,ikk}$:

$$\lambda'_{q+2,ikk} = \int_0^\infty \omega^{q+2} G'_{ik}(\omega)\mathrm{d}\omega \tag{24}$$

which converges for $G_{kk}(\omega)$ of order $\frac{1}{\omega^p}, p>q-3$, for large ω. Next, we show that the modified spectral moments, $\lambda'_{q+2,ikk}$, can be used for the computation of the spectral moments of the response. From Equation (16), (19), and (23), we have

DOI: 10.1002/eqe

$$\int_0^\infty \sum_{i=1}^{n}\sum_{k=1}^{m}\frac{\beta_{ikk}}{\omega_i^2}\omega^{q+2}|H_i(\omega)|^2 G_{kk}(\omega)\mathrm{d}\omega = \int_0^{\omega_o}\sum_{i=1}^{n}\sum_{k=1}^{m}\frac{\beta_{ikk}}{\omega_i^2}\omega^{q+2}G'_{ik}(\omega)\mathrm{d}\omega + \int_{\omega_o}^{\infty}\sum_{i=1}^{n}\sum_{k=1}^{m}\frac{\beta_{ikk}}{\omega_i^2}\omega^{q+2}\left[G'_{ik}(\omega)+\frac{G_{kk}(\omega)}{\omega^4}\right]\mathrm{d}\omega \tag{25}$$

and thus,

$$\int_0^\infty \sum_{i=1}^{n}\sum_{k=1}^{m}\frac{\beta_{ikk}}{\omega_i^2}\omega^{q+2}|H_i(\omega)|^2 G_{kk}(\omega)\mathrm{d}\omega = \int_0^{\infty}\sum_{i=1}^{n}\sum_{k=1}^{m}\frac{\beta_{ikk}}{\omega_i^2}\omega^{q+2}G'_{ik}(\omega)\mathrm{d}\omega + \int_{\omega_o}^{\infty}\sum_{i=1}^{n}\sum_{k=1}^{m}\frac{\beta_{ikk}}{\omega_i^2}\omega^{q+2}\frac{G_{kk}(\omega)}{\omega^4}\mathrm{d}\omega \tag{26}$$

Because of convergence of $\lambda'_{q+2,ikk}$, integration and summation can be interchanged in the first term of the right hand side of (26), thus

$$\int_0^\infty \sum_{i=1}^{n}\sum_{k=1}^{m}\frac{\beta_{ikk}}{\omega_i^2}\omega^{q+2}|H_i(\omega)|^2 G_{kk}(\omega)\mathrm{d}\omega = \sum_{i=1}^{n}\sum_{k=1}^{m}\frac{\beta_{ikk}}{\omega_i^2}\int_0^{\infty}\omega^{q+2}G'_{ik}(\omega)\mathrm{d}\omega + \int_{\omega_o}^{\infty}\frac{1}{\omega^{2-q}}\sum_{k=1}^{m}G_{kk}(\omega)\sum_{i=1}^{n}\frac{\beta_{ikk}}{\omega_i^2}\mathrm{d}\omega \tag{27}$$

We will next show that for any k, $\sum_{i=1}^{n}\frac{\beta_{ikk}}{\omega_i^2}=0$. Because $\beta_{ikl}=\sum_{\substack{j=1\\ j\neq i}}^{n}\left(c_{ki}c_{lj}+c_{li}c_{kj}\right)B_{ij}$, we have that $\sum_{i=1}^{n}\frac{\beta_{ikk}}{\omega_i^2}=2\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}c_{ki}c_{kj}\frac{B_{ij}}{\omega_i^2}$. It suffices then to show that $c_{ki}c_{kj}\frac{B_{ij}}{\omega_i^2}+c_{kj}c_{ki}\frac{B_{ji}}{\omega_j^2}=0$ for any i,j, $j\neq i$. Given that $c_{ki}c_{kj}=c_{kj}c_{ki}$, one must therefore show that $\frac{B_{ij}}{\omega_i^2}+\frac{B_{ji}}{\omega_j^2}=0$. Considering from Equation (8b) that $B_{ij}=-q^2E_{ij}$, then $\frac{B_{ij}}{\omega_i^2}+\frac{B_{ji}}{\omega_j^2}=-\frac{E_{ij}}{\omega_j^2}-\frac{E_{ji}}{\omega_i^2}$. We show in Appendix A that $\frac{E_{ij}}{\omega_j^2}+\frac{E_{ji}}{\omega_i^2}=0$, and hence, $\frac{B_{ij}}{\omega_i^2}+\frac{B_{ji}}{\omega_j^2}=0$. Consequently, the second term in the right hand side of Equation (27) vanishes, similarly to what is shown by Igusa *et al.* [27] for a single ground acceleration, and

$$\int_0^\infty \sum_{i=1}^{n}\sum_{k=1}^{m}\frac{\beta_{ikk}}{\omega_i^2}\omega^{q+2}|H_i(\omega)|^2 G_{kk}(\omega)\mathrm{d}\omega = \sum_{i=1}^{n}\sum_{k=1}^{m}\frac{\beta_{ikk}}{\omega_i^2}\int_0^{\infty}\omega^{q+2}G'_{ik}(\omega)\mathrm{d}\omega \tag{28}$$

Therefore,

$$\int_0^\infty \sum_{i=1}^{n}\sum_{k=1}^{m}\frac{\beta_{ikk}}{\omega_i^2}\omega^{q+2}|H_i(\omega)|^2 G_{kk}(\omega)\mathrm{d}\omega = \sum_{i=1}^{n}\sum_{k=1}^{m}\frac{\beta_{ikk}}{\omega_i^2}\lambda'_{q+2,ikk} \tag{29}$$

which shows that $\lambda'_{q+2,ikk}$ can be used in Equation (19) without modifying the value of λ_q. The spectral moments of the structural response are then given by

$$\lambda_q=\sum_{i=1}^{n}\sum_{k=1}^{m}\left[\alpha_{ikk}\lambda_{q,ikk}+\frac{\beta_{ikk}}{\omega_i^2}\lambda'_{q+2,ikk}\right]+\sum_{i=1}^{n}\sum_{k=1}^{m}\sum_{\substack{l=1\\ k\neq l}}^{m}\left[\alpha_{ikl}\mathrm{Re}\lambda_{q,ikl}+\frac{\beta_{ikl}}{\omega_i^2}\mathrm{Re}\lambda_{q+2,ikl}-\frac{\alpha'_{ikl}}{\omega_i}\mathrm{Im}\lambda_{q+1,ikl}-\frac{\beta'_{ikl}}{\omega_i^3}\mathrm{Im}\lambda_{q+3,ikl}\right] \tag{30}$$

Equation (30) shows that the spectral moments of the dynamic response can be expressed rigorously as a sum of uncoupled spectral moments of modal responses. The correlation between modal responses, for given support motions, is accounted rigorously by the response coefficients $\alpha_{ikl},\beta_{ikl},\alpha'_{ikl}$, and β'_{ikl}, which only depend on structural properties. The space–time correlation of the multi-support earthquake excitation input is considered through the cross-covariance of single modal responses to pairs of support ground motions. The first term in the right hand of Equation (30) represents the contribution from individual modal responses to each support ground motion. The second term takes into account contributions from modal responses to cross-correlated support ground motions.

4. RESPONSE VARIANCE

Among spectral moments, the response variance is fundamental for the development of response spectrum methods. For this reason, we examine next the dynamic response variance, $\sigma_d^2 = \lambda_0$; from Equation (30),

$$\sigma_d^2 = \sum_{i=1}^{n}\sum_{k=1}^{m}\left[\alpha_{ikk}\lambda_{0,ikk} + \frac{\beta_{ikk}}{\omega_i^2}\lambda'_{2,ikk}\right] + \sum_{i=1}^{n}\sum_{k=1}^{m}\sum_{\substack{l=1\\k\neq l}}^{m}\left[\alpha_{ikl}\mathrm{Re}\lambda_{0,ikl} + \frac{\beta_{ikl}}{\omega_i^2}\mathrm{Re}\lambda_{2,ikl} - \frac{\alpha'_{ikl}}{\omega_i}\mathrm{Im}\lambda_{1,ikl} - \frac{\beta'_{ikl}}{\omega_i^3}\mathrm{Im}\lambda_{3,ikl}\right] \quad (31)$$

The spectral moments in the right hand side of Equation (31) can be interpreted in terms of responses of a given mode to pairs of support ground motions. It can be shown that $\lambda_{0,ikl}$ is equal to the covariance between the response displacements Y_{ki} and Y_{li} of the SDOF modal oscillator with natural frequency, ω_i, damping ratio ξ_i, to the support ground accelerations $\ddot{u}_k(t)$ and $\ddot{u}_l(t)$, $\lambda_{0,ikl} = Cov(Y_{ki}, Y_{li})$. Also, it is straightforward to show that $\lambda_{2,ikl}$ represents the covariance between modal velocities $\dot{Y}_{ki}$ and $\dot{Y}_{li}$, $\lambda_{2,ikl} = Cov\left(\dot{Y}_{k,i}, \dot{Y}_{l,i}\right)$. Heredia and Vanmarcke [22] showed that $-\lambda_{1,ikl} = Cov\left(\dot{Y}_{k,i}, , Y_{l,j}\right)$ is the covariance between modal velocity $\dot{Y}_{ki}$ and modal displacement Y_{li}, and $\lambda_{3,ikl} = Cov\left(\ddot{Y}_{k,i}, ,\dot{Y}_{l,j}\right)$ is the covariance between modal acceleration $\ddot{Y}_{ki}$ and modal velocity $\dot{Y}_{li}$. Therefore, the spectral moments represent covariances between single mode responses to pairs of support ground accelerations. Let ρ_{ikl} denote the cross-correlation coefficient between modal displacements Y_{ki} and Y_{li}, and σ_{ik} denote the standard deviation of Y_{ki},

$$\sigma_{ik}^2 = \lambda_{0,ikk} = \int_0^\infty |H_i(\omega)|^2 G_{kk}(\omega)\mathrm{d}\omega \quad (32)$$

so that $\lambda_{0,ikl} = \rho_{ikl}\sigma_{ik}\sigma_{il}$; then,

$$\sigma_d^2 = \sum_{i=1}^{n}\sum_{k=1}^{m}\left[\alpha_{ikk} + \frac{\beta_{ikk}\lambda'_{2,ikk}}{\omega_i^2\lambda_{0,ikk}}\right]\sigma_{ik}^2 + \sum_{i=1}^{n}\sum_{k=1}^{m}\sum_{\substack{l=1\\k\neq l}}^{m}\left[\alpha_{ikl} + \frac{\beta_{ikl}}{\omega_i^2}\frac{\lambda_{2,ikl}}{\lambda_{0,ikl}} - \frac{\alpha'_{ikl}}{\omega_i}\frac{\lambda_{1,ikl}}{\lambda_{0,ikl}} - \frac{\beta'_{ikl}}{\omega_i^3}\frac{\lambda_{3,ikl}}{\lambda_{0,ikl}}\right]\rho_{ikl}\sigma_{ik}\sigma_{il} \quad (33)$$

If we let

$$\Gamma_{ik} = \alpha_{ikk} + \frac{\beta_{ikk}}{\omega_i^2}\frac{\lambda'_{2,ikk}}{\lambda_{0,ikk}} \quad (34a)$$

$$\Theta_{ikl} = \left[\alpha_{ikl} + \frac{\beta_{ikl}}{\omega_i^2}\frac{\lambda_{2,ikl}}{\lambda_{0,ikl}} - \frac{\alpha'_{ikl}}{\omega_i}\frac{\lambda_{1,ikl}}{\lambda_{0,ikl}} - \frac{\beta'_{ikl}}{\omega_i^3}\frac{\lambda_{3,ikl}}{\lambda_{0,ikl}}\right]\rho_{ikl} \quad (34b)$$

the dynamic response variance can be written in a more compact form as follows:

$$\sigma_d^2 = \sum_{i=1}^{n}\sum_{k=1}^{m}\Gamma_{ik}\sigma_{ik}^2 + \sum_{i=1}^{n}\sum_{\substack{k=1\\k\neq l}}^{m}\sum_{l=1}^{m}\Theta_{ikl}\sigma_{ik}\sigma_{il} \quad (35)$$

In Equation (35), the dynamic response is computed based on the analysis of uncoupled modal responses, involving parameters that depend on cross-covariances between single mode responses to

DOI: 10.1002/eqe

pairs of cross-correlated support ground motions and response coefficients that depend solely on structural properties, modal frequencies, and damping. As indicated in Equation (34a) and (34b), parameters Γ_{ik} and Θ_{ikl} are expressed in terms of spectral moments of modal responses, which naturally involve integrations over frequency. The first term in the right hand of Equation (35) accounts for the contribution from individual modal responses to each support ground motion, whereas the second one takes into account contributions from modal responses to cross- correlated support ground motions.

5. GROUND MOTION COHERENCY SPECTRUM

Several functional forms for the coherency spectrum have been proposed based on theoretical and empirical approaches. Der Kiureghian [15] developed a model that accounts for the main effects that give rise to local spatial variation of earthquake ground motion, namely, the incoherence effect, the wave-passage effect, and the site-response effects:

$$\gamma_{kl}(\imath\omega) = \gamma_{kl}(\omega)^{\text{incoherence}} \gamma_{kl}(\imath\omega)^{\text{wave-passage}} \gamma_{kl}(\imath\omega)^{\text{site-effects}} \tag{36}$$

The incoherence effect is due to the scattering of waves in heterogeneous media and their differential super-positioning when arriving from segments of an extended seismic source. The wave-passage effect characterizes spatial variability of the ground motion arising from the difference in the arrival times of waves at separate stations. The site-response component of the coherency spectrum accounts for the spatial variability of the ground motion related to the difference in the local soil conditions at two stations. The incoherence effect is modeled by a real-valued, non-negative decaying function of frequency and distance between supports. Both the wave-passage and site-response effects are modeled by complex-valued phase angle functions. A model that has been used extensively for the incoherence effect is [13]

$$\gamma_{kl}(\omega)^{\text{incoherence}} = \exp\left\{-\left(\frac{\eta\omega d_{kl}}{V_s}\right)^2\right\} \tag{37}$$

where η is an incoherence coefficient, d_{kl} is the distance between support points, and V_s is the shear wave velocity of the ground medium. The wave passage effect is modeled by Der Kiureghian [15]:

$$\gamma_{kl}(\imath\omega)^{\text{wave-passage}} = \exp\left\{-\imath\left(\frac{d_{kl}^L\omega}{V_{\text{app}}}\right)\right\} \tag{38}$$

where d_{kl}^L is the component of support distance along the longitudinal direction of wave propagation, $V_{\text{app}} = \frac{V}{\sin(\psi)}$ is the apparent surface wave velocity, V is the propagation velocity of waves, and ψ is the incidence angle of wave arrival with the normal to the ground surface. The component of coherency spectrum associated with site-response effects is given by Der Kiureghian [15],

$$\gamma_{kl}(\imath\omega)^{\text{site-effects}} = \exp\left\{\imath\left(\tan^{-1}\left(\frac{\text{Im}(H_k(\imath\omega)H_l(-\imath\omega))}{\text{Re}(H_k(\imath\omega)H_l(-\imath\omega))}\right)\right)\right\} \tag{39}$$

where $H_k(\imath\omega)$ is the transfer function for the absolute ground acceleration at the site of the kth support.

6. APPLICATION EXAMPLE

Consider the simply supported two-span beam shown in Figure 1. It has uniform mass and stiffness properties along its longitude: span lengths $L_1 = L_2 = 500\,\text{m}$, $EI = 5.82 \times 10^{10}\,\text{N}\cdot\text{m}^2$ and distributed mass per unit length $m = 232.78\,\text{kg/m}$. For the response analysis, masses are concentrated at mid

DOI: 10.1002/eqe

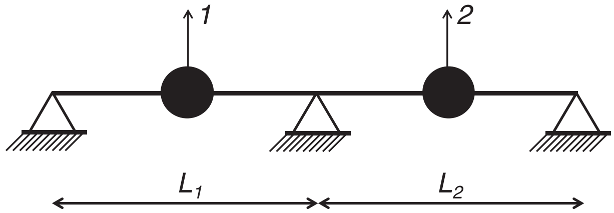


Figure 1. Structural model of two-span beam subjected to seismic ground motion.

spans, and no mass moments of inertia are associated with rotational degrees of freedom. The modal critical damping ratio is assumed to be 5% for all modes. Modal frequencies are $\omega_1 = 9.79$ rad/s and $\omega_2 = 14.81$ rad/s. The beam is subjected to spatially varying earthquake ground accelerations and seismic waves propagate in the direction from support No. 1 to support No. 3. The following responses to the horizontal components of ground accelerations perpendicular to the longitudinal axis of the beam are analyzed: (1) displacement at midspan between supports Nos. 1 and 2; (2) shear force at the center support; (3) bending moment at the center support. The effective influence coefficients, a_k, and effective modal participation factors, $c_{k,i}$, for the three responses of interest are listed in Table I. The response coefficients $\alpha_{ikl}, \beta_{ikl}, \alpha'_{ikl}, \beta'_{ikl}$ for the displacement at midspan are shown in Table II; similarly, these response coefficients are computed for the bending moment and shear force at the center support. All of these coefficients depend only on the structural properties and the response of interest; they are computed once and can then be used for the response analysis to different models of ground motion excitation.

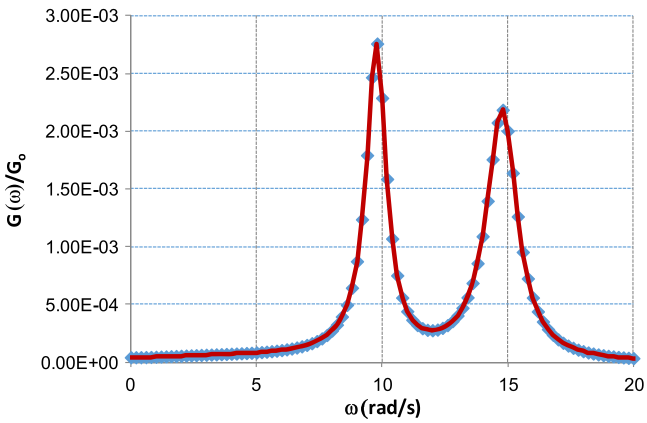
Table I. Effective-influence coefficients and effective modal participation factors.

	Mode 'i'	Support 'k'	Displacement	Bending moment	Shear
a_k	–	1	0.4062	$3.4919 \times 10^{+05}$	$6.9838 \times 10^{+02}$
		2	0.6875	$-6.9838 \times 10^{+05}$	$1.3968 \times 10^{+03}$
		3	−0.0937	$3.4919 \times 10^{+05}$	$6.9838 \times 10^{+02}$
$c_{k,i}$	1	1	0.2500	0.0	$1.3968 \times 10^{+06}$
		2	0.0000	0.0	0.0
		3	−0.2500	0.0	$-1.3968 \times 10^{+06}$
	2	1	0.1563	$7.4830 \times 10^{+05}$	$2.7444 \times 10^{+06}$
		2	0.6875	$3.2922 \times 10^{+06}$	$1.2196 \times 10^{+07}$
		3	0.1563	$7.4830 \times 10^{+05}$	$2.7444 \times 10^{+06}$

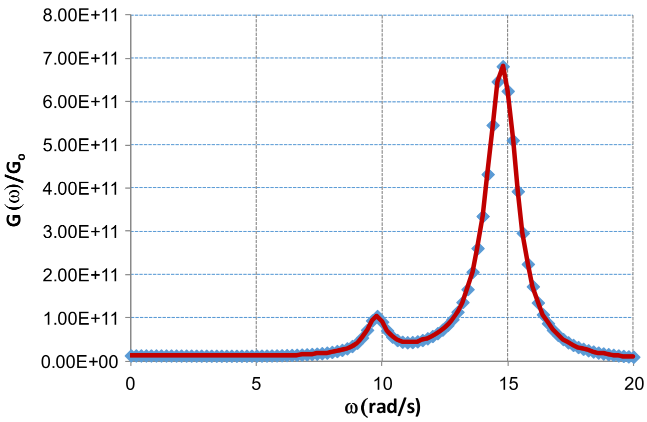
Table II. Displacement response coefficients.

i					1				
k	1	1	1	2	2	2	3	3	3
l	1	2	3	1	2	3	1	2	3
α_{ikl}	0.121	0.129	−0.063	0.129	0	−0.129	−0.063	−0.129	0.004
β_{ikl}	−0.057	−0.126	0	−0.126	0	0.126	0	0.126	0.057
α'_{ikl}	0	0.012	0.005	−0.012	0	0.012	−0.005	−0.012	0
β'_{ikl}	0	−0.025	−0.011	0.025	0	−0.025	0.011	0.025	0
i					2				
k	1	1	1	2	2	2	3	3	3
l	1	2	3	1	2	3	1	2	3
α_{ikl}	−0.103	−0.173	0.024	−0.173	0.473	0.388	0.024	0.388	0.152
β_{ikl}	0.131	0.289	0	0.289	0	−0.289	0	−0.289	−0.131
α'_{ikl}	0	−0.113	−0.052	0.113	0	−0.113	0.052	0.113	0
β'_{ikl}	0	0.085	0.039	−0.085	0	0.085	−0.039	−0.085	0

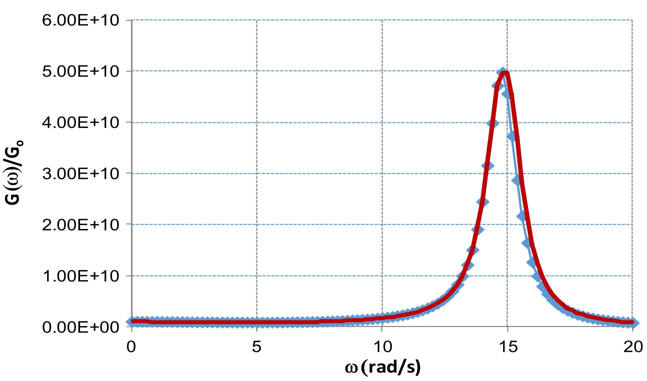
The coherency spectrum model described in Section 5 is used considering $\frac{\eta}{V_s} = 4\times10^{-04}$ s/m and $V_{\text{app}} = 5000$ m/s. First, the case of white noise ground excitation is analyzed, $G_{kk}(\omega) = G_o$. Figure 2 shows a comparison of the auto-power spectra of the structural responses computed using the analytical expression in Equation (6) and that using the partial fraction decomposition, as given in Equation (14). It can be seen that for all of the responses, the auto-power spectra using the partial fraction expansion coincides precisely with the theoretical one. Peaks of the auto-power spectra are observed at the modal frequencies of the structure. These results show that the partial fraction decomposition works correctly and that the auto-power spectrum of the structural response can be



(a) Displacement



(b) Shear



(c) Bending Moment

Figure 2. Power spectra of structural responses (dotted-line: exact solution; full-line: partial-fraction solution); white noise model.

DOI: 10.1002/eqe

computed in terms of sums of uncoupled squared norms of modal transfer functions. The spectral moments of modal responses, as defined in Equations (18) and (24), are listed in Table III. As noted, they are computed for each individual mode. The first three spectral moments of the dynamic component of the displacement, bending moment, and shear responses were computed using the formulation proposed in this paper. The spectral moments of these responses were also computed using the analytical solution for the auto-power spectrum of the dynamic response as given in Equation (6), which we term here as the 'exact response'. As shown in Table IV, the results obtained with the formulation proposed in this paper coincide with the exact response in all cases. Numerical errors listed in Table IV are negligible; they are less than 0.04% in all cases, thus illustrating how the proposed method yields the correct results. In Table V, spectral moments of the modal responses and of the structural response are given as function of the arbitrary frequency ω_o, which is used in the definition of the modified spectral density $G'_{ik}(\omega)$, Equation (23). The values of the modified spectral moments $\lambda'_{q+2,ikk}$ vary with the choice of ω_o; however, as proven in Equation (29), the value of the structural response is not influenced by the choice of ω_o. The original moments can be computed only up to order 2, and higher order moments do not exist, whereas for the modified spectral moments, orders up to 4 do exist and can be computed.

Next, the well-known modified Kanai–Tajimi auto power spectrum is used to model ground accelerations on soft and firm soil conditions:

$$G_{kk}(\omega) = \frac{1 + 4\xi_{fk}(\omega/\omega_{fk})^2}{\left[1 - (\omega/\omega_{fk})^2\right]^2 + 4\xi_{fk}^2(\omega/\omega_{fk})^2} \frac{(\omega/\omega_{gk})^4}{\left[1 - (\omega/\omega_{gk})^2\right]^2 + 4\xi_{gk}^2(\omega/\omega_{gk})^2} G_{ok} \tag{40}$$

Table III. Spectral moments of modal responses; white noise model.

Order (q)	ω_i (rad/s)	$\lambda'_{q+2,ikk}$ ($\omega_o = 1000$)	$\lvert x_{kl}\rvert$ (m)	Re $\lambda_{q,ikl}$	Im $\lambda_{q,ikl}$
0	9.79	3.20	0	3.34×10^{-02}	0.00
			500	1.64×10^{-03}	-1.02×10^{-03}
			1000	4.81×10^{-04}	-1.50×10^{-04}
	14.81	2.12	0	9.67×10^{-03}	0.00
			500	1.96×10^{-04}	-6.87×10^{-05}
			1000	8.87×10^{-05}	-2.64×10^{-05}
1	9.79	$3.96\times10^{+01}$	0	3.17×10^{-01}	0.00
			500	7.94×10^{-03}	-7.61×10^{-03}
			1000	6.79×10^{-04}	-3.53×10^{-04}
	14.81	$3.87\times10^{+01}$	0	1.39×10^{-01}	0.00
			500	5.96×10^{-04}	-3.67×10^{-04}
			1000	1.20×10^{-04}	-5.89×10^{-05}
2	9.79	$2.30\times10^{+03}$	0	3.21	0.00
			500	5.59×10^{-02}	-6.39×10^{-02}
			1000	1.50×10^{-03}	-1.06×10^{-03}
	14.81	$2.46\times10^{+03}$	0	2.12	0.00
			500	2.87×10^{-03}	-2.58×10^{-03}
			1000	2.55×10^{-04}	-1.68×10^{-04}
3	9.79	(*)	500	4.54×10^{-01}	-5.68×10^{-01}
			1000	4.21×10^{-03}	-3.76×10^{-03}
	14.81	(*)	500	1.79×10^{-02}	-2.20×10^{-02}
			1000	6.88×10^{-04}	-5.63×10^{-04}
4	9.79	(*)	500	3.94	-5.21
			1000	1.39×10^{-02}	-1.52×10^{-02}
	14.81	(*)	500	1.32×10^{-01}	-2.16×10^{-01}
			1000	2.18×10^{-03}	-2.14×10^{-03}
5	9.79	(*)	500	$3.56\times10^{+01}$	$-4.88\times10^{+01}$
			1000	5.13×10^{-02}	-6.85×10^{-02}
	14.81	(*)	500	1.12	-2.35
			1000	7.74×10^{-03}	-8.96×10^{-03}

*Note: not required and do not exist.

DOI: 10.1002/eqe

Table IV. Spectral moments of dynamic response; white noise model.

	Displacement		
Solution	Zero moment $/G_o$ (s^3)	First moment $/G_o$ (s^2)	Second moment $/G_o$ (s)
Exact	9.212×10^{-03}	1.119×10^{-01}	1.505
Proposed	9.216×10^{-03}	1.119×10^{-01}	1.506
error (%)	0.04	0.06	0.07
	Shear		
Solution	Zero moment$/G_o$ (N^2s^3/m^2)	First moment$/G_o$ (N^2s^2/m^2)	Second moment$/G_o$ (N^2s/m^2)
Exact	$1.736\times10^{+12}$	$2.402\times10^{+13}$	$3.600\times10^{+14}$
Proposed	$1.736\times10^{+12}$	$2.402\times10^{+13}$	$3.600\times10^{+14}$
Error (%)	0.002	0.002	0.002
	Bending moment		
Solution	Zero moment$/G_o$ (N^2s^3)	First moment$/G_o$ (N^2s^2)	Second moment$/G_o$ (N^2s)
Proposed	$1.176\times10^{+11}$	$1.665\times10^{+12}$	$2.539\times10^{+13}$
Exact	$1.176\times10^{+11}$	$1.665\times10^{+12}$	$2.539\times10^{+13}$
Error (%)	0.004	0.004	0.004

Table V. Effect of ω_o on modified modal spectral moments and on structural response; white noise model.

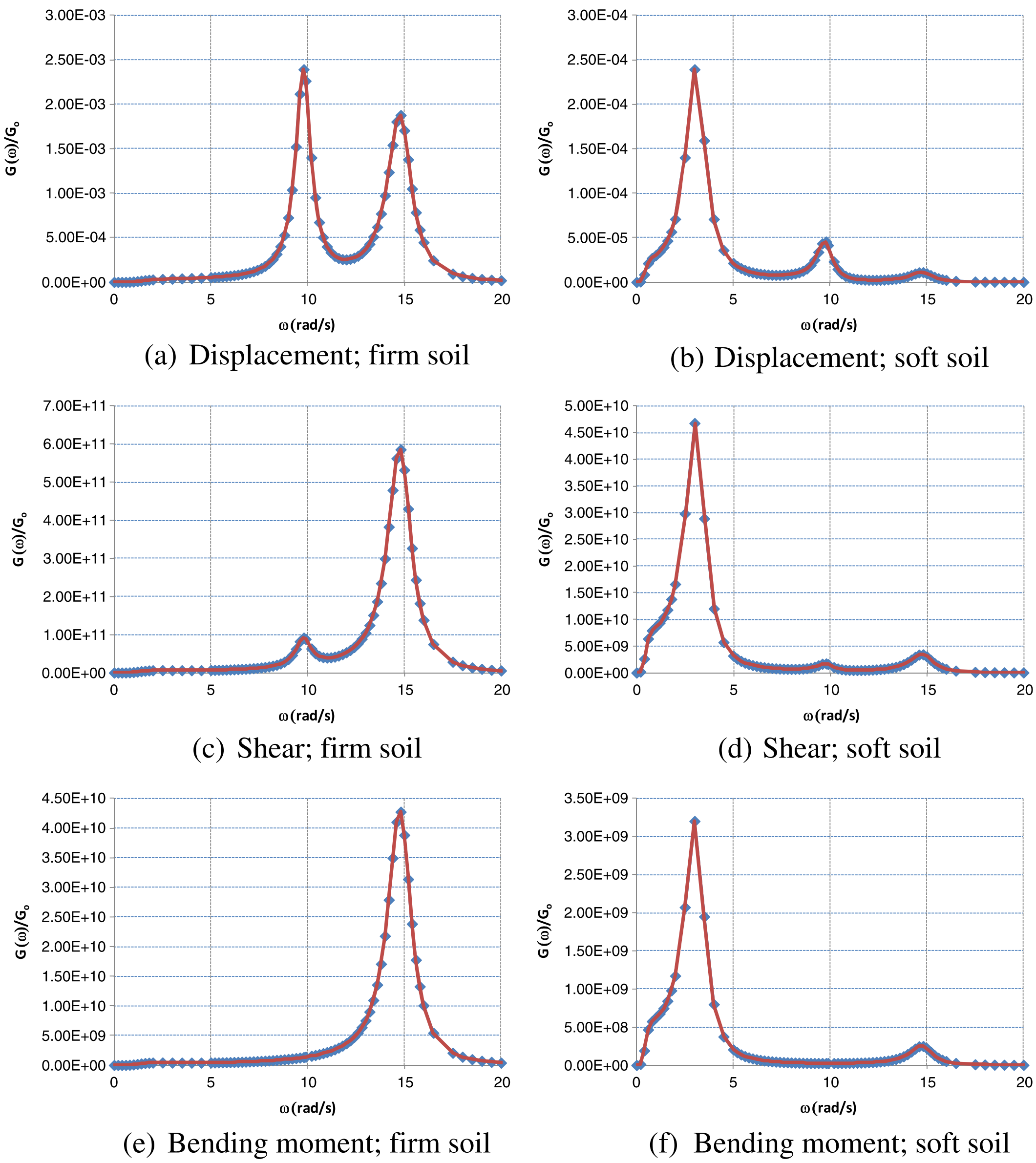
			$\lambda'_{q+2,ikk}$		
Mode	q	$\lambda_{q,ikk}$	$\omega_o=10$ rad/s	$\omega_o=100$ rad/s	$\omega_o=1000$ rad/s
$\omega_i=9.79$ rad/s	0	3.339×10^{-02}	3.006	3.186	3.204
	1	3.172×10^{-01}	30.342	34.947	39.552
	2	3.206	324.733	504.733	2304.733
$\omega_i=14.81$ rad/s	0	9.665×10^{-03}	1.920	2.100	2.118
	1	1.387×10^{-01}	29.515	34.120	38.725
	2	2.120	480.714	660.714	2460.714
Displacement	Zero moment$/G_o$ (s^3)	9.212×10^{-03}	9.216×10^{-03}	9.216×10^{-03}	9.216×10^{-03}
spectral moments	First moment$/G_o$ (s^2)	1.119×10^{-01}	1.119×10^{-01}	1.119×10^{-01}	1.119×10^{-01}
	Second moment$/G_o$ (s)	1.505	1.506	1.506	1.506

where the model parameters ω_{fk}, ω_{gk}, and ξ_{fk}, ξ_{gk} can be interpreted as the natural frequencies and dampings of a soil layer, and G_{ok} as the intensity of a white noise ground acceleration at the base of the soil deposit. Table VI lists the values of the parameters used here for modeling ground motions in firm and soft soil conditions. Figure 3 shows a comparison of the auto-power spectra of the structural responses computed using the analytical expression in Equation (6) and that using the partial fraction decomposition, as given in Equation (14), for firm and soft soil conditions. It can be seen that the theoretical auto-power spectra can be reproduced using the partial fraction decomposition for all of the structural responses in both soil conditions. Peaks of the auto-power spectra are observed at the modal frequencies of the structure; in the case of soft soil conditions, a major peak is also observed at the dominant frequency of ground motion, ω_f. These results show again that the partial fraction decomposition works correctly for the computation of the auto-power spectrum of the structural response. The spectral moments of modal responses are listed in Tables VII and VIII. The first three spectral moments of the dynamic component of the displacement, shear force, and bending moment responses obtained using the exact solution and the formulation proposed in this paper are listed in Table IX. The results obtained with the proposed formulation

Table VI. Parameters of ground acceleration Kanai–Tajimi power spectrum model.

Type of soil	ω_f (rad/s)	ξ_f	ω_g (rad/s)	ξ_g
Soft	π	0.2	0.5	0.6
Stiff	15	0.6	1.5	0.6

DOI: 10.1002/eqe



(a) Displacement; firm soil (b) Displacement; soft soil

(c) Shear; firm soil (d) Shear; soft soil

(e) Bending moment; firm soil (f) Bending moment; soft soil

Figure 3. Power spectra of structural responses (dotted-line: exact solution; full-line: partial-fraction solution); Kanai–Tajimi model.

coincide extremely well with the exact response in all cases. As shown in Table IX, numerical errors are negligible; they are less than 0.03%, which verifies that the proposed method allows computing the exact solution correctly. Notice that in the case of the Kanai–Tajimi model, $G_{kk}(\omega)\rightarrow\frac{4\xi_f\omega_f^2}{\omega^2}$ for large ω; hence, $G_{kk}(\omega)$ is of order $p=2$, spectral moments $\lambda_{q+2,ikk}$ can be computed for $q<3$, and the first three spectral moments of the structural response can be obtained. Therefore, the same response is estimated if one uses the original spectral moments $\lambda_{q+2,ikk}$ instead of the modified spectral moments $\lambda'_{q+2,ikk}$.

The spectral moments of the total response were computed considering the dynamic response component and the contributions from the pseudo-static component and from the cross-correlation between them; results are listed in Table X. Figure 4 shows the relative weight of the contributions from the pseudo-static, dynamic, and cross-correlation components to the first three spectral moments of displacement, shear, and bending moment, expressed as a percentage of the total response; when the contribution from the cross-correlation is negative, it is shown on the negative side of the vertical axes. The pseudo-static component is dominant in the estimation of all spectral

DOI: 10.1002/eqe

Table VII. Spectral moments of modal responses; Kanai–Tajimi model.

			Firm soil		Soft soil	
Order (q)	ω_i (rad/s)	$\lvert x_{kl} \rvert$ (m)	Re $\lambda_{q,ikl}$	Im $\lambda_{q,ikl}$	Re $\lambda_{q,ikl}$	Im $\lambda_{q,ikl}$
0	9.79	0	5.58×10^{-02}	0.00	4.86×10^{-03}	0.00
		500	1.86×10^{-03}	-1.53×10^{-03}	2.46×10^{-03}	-7.26×10^{-04}
		1000	2.47×10^{-04}	-1.32×10^{-04}	9.55×10^{-04}	-4.60×10^{-04}
	14.81	0	1.61×10^{-02}	0.00	7.39×10^{-04}	0.00
		500	1.74×10^{-04}	-8.73×10^{-05}	4.19×10^{-04}	-1.17×10^{-04}
		1000	4.39×10^{-05}	-2.25×10^{-05}	1.71×10^{-04}	-8.02×10^{-05}
1	9.79	0	5.38×10^{-01}	0.00	2.20×10^{-02}	0.00
		500	1.16×10^{-02}	-1.22×10^{-02}	6.93×10^{-03}	-2.46×10^{-03}
		1000	5.74×10^{-04}	-3.73×10^{-04}	2.06×10^{-03}	-1.19×10^{-03}
	14.81	0	2.29×10^{-01}	0.00	3.37×10^{-03}	0.00
		500	7.31×10^{-04}	-5.27×10^{-04}	1.13×10^{-03}	-3.63×10^{-04}
		1000	9.91×10^{-05}	-6.11×10^{-05}	3.62×10^{-04}	-2.05×10^{-04}
2	9.79	0	5.36	0.00	1.41×10^{-01}	0.00
		500	8.79×10^{-02}	-1.05×10^{-01}	2.29×10^{-02}	-9.95×10^{-03}
		1000	1.55×10^{-03}	-1.22×10^{-03}	5.25×10^{-03}	-3.41×10^{-03}
	14.81	0	3.37	0.00	2.75×10^{-02}	0.00
		500	3.97×10^{-03}	-3.99×10^{-03}	3.46×10^{-03}	-1.25×10^{-03}
		1000	2.59×10^{-04}	-1.90×10^{-04}	9.06×10^{-04}	-5.76×10^{-04}
3	9.79	500	7.38×10^{-01}	-9.48×10^{-01}	8.86×10^{-02}	-4.98×10^{-02}
		1000	4.75×10^{-03}	-4.56×10^{-03}	1.47×10^{-02}	-1.04×10^{-02}
	14.81	500	2.65×10^{-02}	-3.58×10^{-02}	1.18×10^{-02}	-4.87×10^{-03}
		1000	7.67×10^{-04}	-6.70×10^{-04}	2.50×10^{-03}	-1.73×10^{-03}
4	9.79	500	6.53	−8.79	4.17×10^{-01}	-3.11×10^{-01}
		1000	1.64×10^{-02}	-1.90×10^{-02}	4.41×10^{-02}	-3.35×10^{-02}
	14.81	500	2.07×10^{-01}	-3.64×10^{-01}	4.44×10^{-02}	-2.22×10^{-02}
		1000	2.55×10^{-03}	-2.62×10^{-03}	7.40×10^{-03}	-5.49×10^{-03}
5	9.79	500	$5.96\times10^{+01}$	$-8.29\times10^{+01}$	2.43	−2.31
		1000	6.24×10^{-02}	-8.81×10^{-02}	1.39×10^{-01}	-1.14×10^{-01}
	14.81	500	1.83	−4.04	1.90×10^{-01}	-1.24×10^{-01}
		1000	9.34×10^{-03}	-1.12×10^{-02}	2.31×10^{-02}	-1.83×10^{-02}

Table VIII. Modified spectral moments of modal responses; Kanai–Tajimi model.

		$\lambda'_{q+2,ikk}$	
Order (q)	ω_i (rad/s)	Firm soil	Soft soil
0	9.79	3.45	−7.49
	14.81	1.47	−7.60
1	9.79	$4.66\times10^{+01}$	$-1.07\times10^{+01}$
	14.81	$4.23\times10^{+01}$	$-1.15\times10^{+01}$
2	9.79	$5.09\times10^{+02}$	$-1.79\times10^{+01}$
	14.81	$6.99\times10^{+02}$	$-2.36\times10^{+01}$

moments of the displacement response only in soft soil. However, in firm soil conditions, the dynamic component may me relevant as well and cannot be neglected; it is also observed that in this case, the dynamic component becomes more important for higher order moments of the displacement response. The dynamic component is dominant in the computation of all three spectral moments of the shear response in both soil conditions, accounting for about 99% of the total response. In the case of bending moment, the dynamic component controls the response only in the case of firm soil. In soft soil, the contributions of both the pseudo-static and dynamic components, as well as their cross-correlations, are relevant for the computation of the response; it is noted that the relative weight of the dynamic component becomes more important for higher order spectral moments of the bending response. In summary, the dynamic response was found to be relevant in the estimation of the first

DOI: 10.1002/eqe

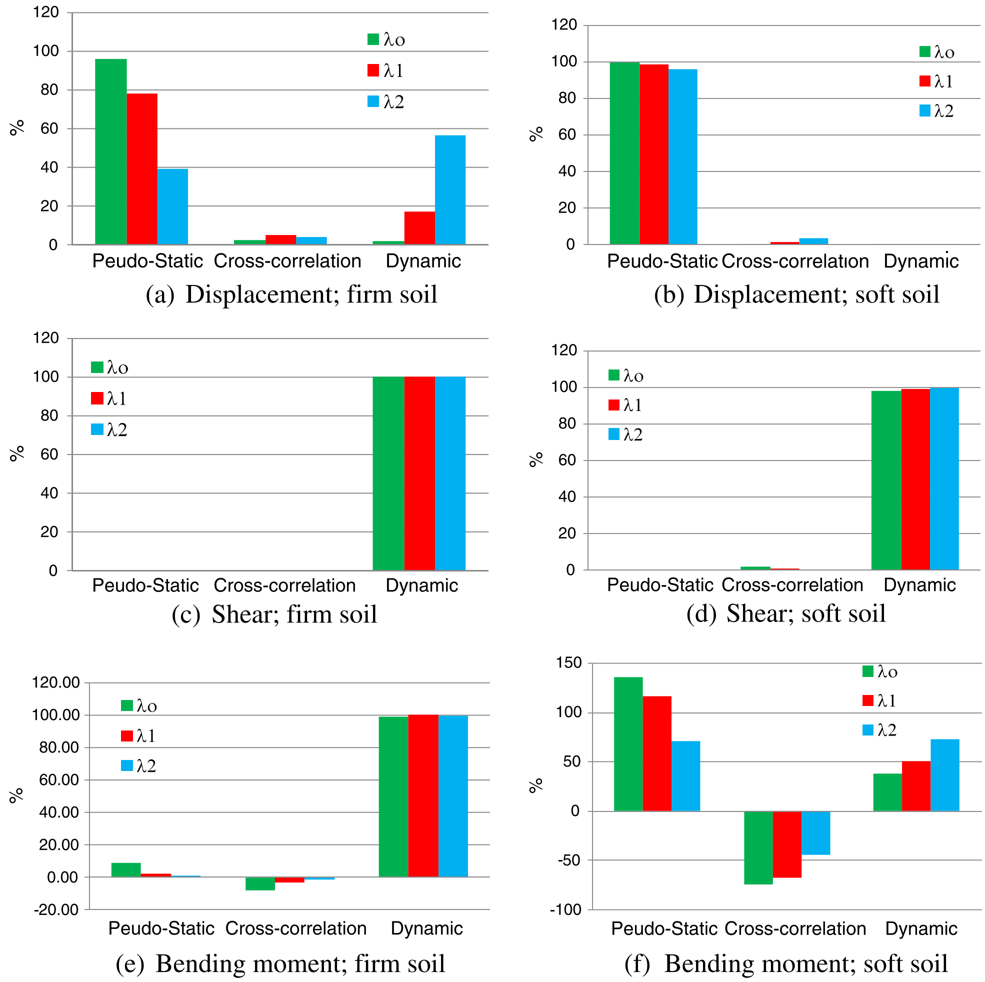
Table IX. Spectral moments of dynamic response; Kanai–Tajimi model.

	Displacement					
	Firm soil			Soft soil		
Solution	Zero moment/G_o (s^3)	First moment/G_o (s^2)	Second moment/G_o (s)	Zero moment/G_o (s^3)	First moment/G_o (s^2)	Second moment/G_o (s)
Exact	1.534×10^{-02}	1.866×10^{-01}	2.424	1.058×10^{-03}	4.734×10^{-03}	3.266×10^{-02}
Proposed	1.535×10^{-02}	1.867×10^{-01}	2.426	1.057×10^{-03}	4.734×10^{-03}	3.267×10^{-02}
Error (%)	0.05	0.06	0.07	0.06	0.007	0.03
Solution	Shear					
	Firm soil			Soft soil		
	Zero moment/G_o (N^2s^3/m^2)	First moment/G_o (N^2s^2/m^2)	Second moment/G_o (N^2s/m^2)	Zero moment/G_o (N^2s^3/m^2)	First moment/G_o (N^2s^2/m^2)	Second moment/G_o (N^2s/m^2)
Exact	$2.865\times10^{+12}$	$3.970\times10^{+13}$	$5.731\times10^{+14}$	$1.945\times10^{+11}$	$7.842\times10^{+11}$	$5.491\times10^{+12}$
Proposed	$2.865\times10^{+12}$	$3.970\times10^{+13}$	$5.731\times10^{+14}$	$1.945\times10^{+11}$	$7.842\times10^{+11}$	$5.491\times10^{+12}$
Error (%)	0.002	0.002	0.002	0.0004	0.0009	0.001
Solution	Bending moment					
	Firm soil			Soft soil		
	Zero moment/G_o (N^2s^3)	First moment/G_o (N^2s^2)	Second moment/G_o (N^2s)	Zero moment/G_o (N^2s^3)	First moment/G_o (N^2s^2)	Second moment/G_o (N^2s)
Exact	$1.937\times10^{+11}$	$2.749\times10^{+12}$	$4.035\times10^{+13}$	$1.316\times10^{+10}$	$5.182\times10^{+10}$	$3.636\times10^{+11}$
Proposed	$1.937\times10^{+11}$	$2.749\times10^{+12}$	$4.035\times10^{+13}$	$1.316\times10^{+10}$	$5.182\times10^{+10}$	$3.636\times10^{+11}$
Error (%)	0.004	0.004	0.004	0.009	0.007	0.006

DOI: 10.1002/eqe

Table X. Components of spectral moments of total response; Kanai–Tajimi model.

	Displacement					
	Firm soil			Soft soil		
Response component	Zero moment/G_o (s^3)	First moment/G_o (s^2)	Second moment/G_o (s)	Zero moment/G_o (s^3)	First moment/G_o (s^2)	Second moment/G_o (s)
Pseudo-static	0.776	0.857	1.671	$2.211\times10^{+01}$	9.088	7.393
Cross-correl.	0.174×10^{-01}	0.544×10^{-01}	0.175	7.277×10^{-02}	1.160×10^{-01}	2.785×10^{-01}
Dynamic	0.153×10^{-01}	0.186	2.424	1.058×10^{-03}	4.734×10^{-03}	3.266×10^{-02}
Total response	0.809	1.098	4.270	$2.218\times10^{+01}$	9.209	7.704
Response component	Shear					
	Firm soil			Soft soil		
	Zero moment/G_o (N^2s^3/m^2)	First moment/G_o (N^2s^2/m^2)	Second moment/G_o (N^2s/m^2)	Zero moment/G_o (N^2s^3/m^2)	First moment/G_o (N^2s^2/m^2)	Second moment/G_o (N^2s/m^2)
Pseudo-static	$5.745\times10^{+06}$	$6.003\times10^{+06}$	$1.067\times10^{+07}$	$1.706\times10^{+08}$	$6.853\times10^{+07}$	$5.293\times10^{+07}$
Cross-correl.	$6.704\times10^{+08}$	$2.035\times10^{+09}$	$8.006\times10^{+09}$	$3.155\times10^{+09}$	$4.576\times10^{+09}$	$1.025\times10^{+10}$
Dynamic	$2.865\times10^{+12}$	$3.970\times10^{+13}$	$5.731\times10^{+14}$	$1.945\times10^{+11}$	$7.842\times10^{+11}$	$5.491\times10^{+12}$
Total response	$2.866\times10^{+12}$	$3.970\times10^{+13}$	$5.731\times10^{+14}$	$1.979\times10^{+11}$	$7.889\times10^{+11}$	$5.502\times10^{+12}$
Response component	Bending moment					
	Firm soil			Soft soil		
	Zero moment/G_o (N^2s^3)	First moment/G_o (N^2s^2)	Second moment/G_o (N^2s)	Zero moment/G_o (N^2s^3)	First moment/G_o (N^2s^2)	Second moment/G_o (N^2s)
Pseudo-static	$1.751\times10^{+10}$	$7.160\times10^{+10}$	$4.389\times10^{+11}$	$4.658\times10^{+10}$	$1.192\times10^{+11}$	$3.516\times10^{+11}$
Cross-correl.	$-1.605\times10^{+10}$	$-8.762\times10^{+10}$	$-4.434\times10^{+11}$	$-2.550\times10^{+10}$	$-6.901\times10^{+10}$	$-2.184\times10^{+11}$
Dynamic	$1.937\times10^{+11}$	$2.749\times10^{+12}$	$4.035\times10^{+13}$	$1.316\times10^{+10}$	$5.182\times10^{+10}$	$3.636\times10^{+11}$
Total response	$1.952\times10^{+11}$	$2.733\times10^{+12}$	$4.034\times10^{+13}$	$3.424\times10^{+10}$	$1.020\times10^{+11}$	$4.968\times10^{+11}$



(a) Displacement; firm soil
(b) Displacement; soft soil
(c) Shear; firm soil
(d) Shear; soft soil
(e) Bending moment; firm soil
(f) Bending moment; soft soil

Figure 4. Contributions from response components to spectral moments of structural response; Kanai–Tajimi model.

spectral moments of the displacement, shear, and bending moment response under firm soil and soft soil conditions. It is therefore necessary to assess precisely the dynamic component to characterize correctly the total response.

7. CONCLUSIONS

A formulation was developed for modal response analysis of multi-support structures using a random vibration approach. The spectral moments of dynamic response are expressed rigorously as the sum of spectral moments of uncoupled SDOF modal responses. The method accounts fully for the cross-correlation between modal responses and for the space-time cross-correlation between support ground motions. The correlation between modal responses, for given support motions, is rigorously accounted for by the response coefficients derived from the partial fractions approach which depend only on structural properties. The space–time correlation of the multi-support earthquake excitation input is considered through cross-covariances of single modal responses to pairs of support ground motions. Convergence of modal spectral moments has been analyzed in detail and a general formulation implemented, which assures compatibility between existence of spectral moments of the structural response and the spectral moments of modal responses required by the formulation. The dynamic response variance was expressed explicitly in terms of the contribution from the modal responses to individual support ground motions and the contribution from modal responses to

DOI: 10.1002/eqe

cross-correlated pairs of support ground motions. An advantage of the formulation advanced here is that the computation of the dynamic response can proceed on the basis of mode by mode uncoupled analyses. It can be extended into a response spectrum method in which responses of single modes can be assessed first and then the total response is obtained by summing up the contributions of individual modes as in classical modal analysis. An example of a two-span beam was used to illustrate the application and performance of the formulation. Well-known and established models were used for the ground motion power spectra, such as white noise and Kanai–Tajimi for soft and firm soil conditions, and for the coherency spectra, accounting for wave incoherence and passage effects. The first three spectral moments of structural responses of interest were analyzed, finding negligible numerical errors and showing that the formulation performs correctly and yields the same results as the exact solution.

APPENDIX A

Inverting subscripts i and j in Equation (8a), one obtains for E_{ji},

$$E_{ji} = \frac{r^2\left\{\left[4\xi_i\left(r\xi_j-\xi_i\right)+1-r^2\right]\left(r^4-1\right)-2\left(r^2-1\right)\left[\left(1-2\xi_i{}^2\right)-r^2\left(1-2\xi_j{}^2\right)\right]\right\}}{4r^2\left[\left(1-2\xi_i{}^2\right)r^2-\left(1-2\xi_j{}^2\right)\right]\left[\left(1-2\xi_j{}^2\right)r^2-\left(1-2\xi_i{}^2\right)\right]-\left(1-r^4\right)^2} \tag{A1}$$

Then,

$$\frac{E_{ji}}{\omega_i^2} = \frac{1}{\omega_j^2}\frac{\left\{\left[4\xi_i\xi_j r-4\xi_i^2+1-r^2\right]\left(r^4-1\right)-2\left(1-r^2\right)\left[r^2\left(1-2\xi_j{}^2\right)-\left(1-2\xi_i{}^2\right)\right]\right\}}{4r^2\left[\left(1-2\xi_i{}^2\right)r^2-\left(1-2\xi_j{}^2\right)\right]\left[\left(1-2\xi_j{}^2\right)r^2-\left(1-2\xi_i{}^2\right)\right]-\left(1-r^4\right)^2} \tag{A2}$$

From Equation (8a),

$$\frac{E_{ij}}{\omega_j^2} = -\frac{1}{\omega_j^2}\frac{\left\{\left[4r\xi_i\xi_j-4r^2\xi_j^2+r^2-1\right]\left(r^4-1\right)+2r^2\left(1-r^2\right)\left[r^2\left(1-2\xi_j{}^2\right)-\left(1-2\xi_i{}^2\right)\right]\right\}}{4r^2\left[\left(1-2\xi_i{}^2\right)r^2-\left(1-2\xi_j{}^2\right)\right]\left[\left(1-2\xi_j{}^2\right)r^2-\left(1-2\xi_i{}^2\right)\right]-\left(1-r^4\right)^2} \tag{A3}$$

Summing up (A2) and (A3) yields

$$\frac{E_{ij}}{\omega_j^2}+\frac{E_{ji}}{\omega_i^2} = \frac{\left(r^4-1\right)}{\omega_j^2}\left\{\frac{2\left[r^2\left(1-2\xi_j{}^2\right)-\left(1-2\xi_i{}^2\right)\right]-\left[4\xi_i^2-4r^2\xi_j^2+2r^2-2\right]}{4r^2\left[\left(1-2\xi_i{}^2\right)r^2-\left(1-2\xi_j{}^2\right)\right]\left[\left(1-2\xi_j{}^2\right)r^2-\left(1-2\xi_i{}^2\right)\right]-\left(1-r^4\right)^2}\right\} \tag{A.4}$$

It can easily be shown that the numerator between brackets in Equation (A.4) is equal to zero,

$$2\left[r^2\left(1-2\xi_j{}^2\right)-\left(1-2\xi_i{}^2\right)\right]-\left[4\xi_i^2-4r^2\xi_j^2+2r^2-2\right]=0 \tag{A.5}$$

thus showing that

$$\frac{E_{ij}}{\omega_j^2}+\frac{E_{ji}}{\omega_i^2}=0 \tag{A.6}$$

DOI: 10.1002/eqe

ACKNOWLEDGEMENTS

Research conducted by the second author was supported by the Dirección de Gestión de la Investigación of the Pontificia Universidad Católica del Perú under Project No. 0201-2011.

REFERENCES

1. Zerva A. *Spatial Variation of Seismic Ground Motions: Modeling and Engineering Applications*. CRC Press, Taylor and Francis Group: Boca Raton, 2009.
2. Zhang YH, Li QS, Lin JH, Williams FW. Random vibration analysis of long-span structures subjected to spatially varying ground motions. *Soil Dynamics and Earthquake Engineering* 2009; **29**(4):620–629.
3. Su L, Dong S, Kato S. A new average response spectrum method for linear response analysis of structures to spatial earthquake ground motion. *Engineering Structures* 2006; **28**(13):1835–1842.
4. Lupoi A, Franchin P, Pinto PE, Monti G. Seismic design of bridges accounting for spatial variability of ground motion. *Earthquake Engineering and Structural Dynamics* 2005; **34**:327–348.
5. Harichandran RS, Hawwari A, Sweidan BN. Response of long-span bridges to spatially varying ground motion. *Journal of Structural Engineering, ASCE* 1996; **122**(5):476–484.
6. Heredia-Zavoni E, Vanmarcke EH. Random-vibration-based response spectrum method for multi-support structural systems. *Journal of Seismology and Earthquake Engineering* 1998; **1**: 35–50.
7. Chen M-T, Harichandran RS. Sensitivity of earth dam seismic response to ground motion coherency. *Geotechnical Earthquake Engineering and Soil Dynamics III*, Dakoulas P *et al.* (eds.). Geotechnical Special Publication No. 75, ASCE: Reston, 1998.
8. Yamamura N, Tanaka H. Response analysis of flexible MDF systems for multiple-support seismic excitations. *Earthquake Engineering and Structural Dynamics* 1990; **19**:345–357.
9. Harichandran R, Wang W. Response of indeterminate two-span beam to spatially varying seismic excitation. *Earthquake Engineering and Structural Dynamics* 1990; **19**:173–187.
10. Zerva A, Ang AH-S, Wen YK. Lifeline response to spatially variable ground motions. *Earthquake Engineering and Structural Dynamics* 1988; **16**:361–379.
11. Hindy A, Novak M. Pipeline response to random ground motion. *Journal of Engineering Mechanics, ASCE* 1980; **106**(EM2):339–360.
12. Harichandran RS, Vanmarcke EH. Stochastic variation of earthquake ground motion in space and time. *Journal of Engineering Mechanics, ASCE* 1986; **112**(2):154-174.
13. Luco JE, Wong HL. Response of a rigid foundation to a spatially random ground motion. *Earthquake Engineering and Structural Dynamics* 1986; **14**:891–908.
14. Abrahamson NA. Generation of spatially incoherent strong motion time histories. *Proceedings of the 10th World Conference on Earthquake Engineering*, Barcelona, 1992; **10**: 845–850.
15. Der Kiureghian A. A coherency model for spatially varying ground motion. *Earthquake Engineering and Structural Dynamics* 1996; **25**:99–111.
16. Vanmarcke EH, Heredia-Zavoni E, Fenton GA. Conditional simulation of spatially correlated earthquake ground motion. *Journal of Engineering Mechanics, ASCE* 1993; **119**:2333–2352.
17. Heredia-Zavoni E, Santa-Cruz S. Conditional simulation of a class of non-stationary space–time random fields. *Journal of Engineering Mechanics ASCE* 2000; **126**(4):398–404.
18. Liao S, Zerva A. Physically compliant, conditionally simulated spatially variable seismic ground motions for performance-based design. *Earthquake Engineering and Structural Dynamics* 2006; **35** (7):891–919.
19. Shama A. Simplified procedure for simulating spatially correlated earthquake ground motions. *Engineering Structures* 2007; **29**(2):248–258.
20. Cacciola P, Deodatis G. A method for generating fully non-stationary and spectrum-compatible ground motion vector processes. *Soil Dynamics and Earthquake Engineering* 2011; **31**(3):351–360.
21. Der Kiureghian A, Neuenhofer A. Response spectrum method for multiple support seismic excitation. *Earthquake Engineering and Structural Dynamics* 1992; **21**:713–740.
22. Heredia-Zavoni E, Vanmarcke EH. Seismic random vibration analysis of multi-support structural systems. *Journal of Engineering Mechanics ASCE* 1994; **120**:1107–1128.
23. Konakli K, Der Kiureghian A. Extended MSRS rule for seismic analysis of bridges subjected to differential support motions. *Earthquake Engineering and Structural Dynamics* 2011; **40**(12):1315–1335.
24. Wang Z, Der Kiureghian A. Multiple-support response spectrum analysis using load-dependent Ritz vectors. *Earthquake Engineering and Structural Dynamics* 2014; **43**:2283–2297.
25. Heredia-Zavoni E. The complete SRSS modal combination rule. *Earthquake Engineering and Structural Dynamics* 2011; **40**:1181–1196.
26. Der Kiureghian A, Neuenhofer A. Response spectrum method for multiple-support seismic excitation. Report No. UCB/EERC-91/08, Earthquake Engineering Research Center 1991.
27. Igusa T, Der Kiureghian A, Sackman JL. Modal decomposition method for stationary response of non-classically damped systems. *Earthquake Engineering and Structural Dynamics* 1984; **12**:121–136.

DOI: 10.1002/eqe